Write your name here

Surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number | | | | | |

Candidate Number | | | | |

Core Mathematics C34

Advanced

Wednesday 8 November 2017 – Morning
Time: 2 hours 30 minutes

Paper Reference
WMA02/01

You must have:
Mathematical Formulae and Statistical Tables (Blue)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 125.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

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1. $f(x) = x^5 + x^3 - 12x^2 - 8, \qquad x \in \mathbb{R}$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt[3]{\frac{4(3x^2 + 2)}{x^2 + 1}}$$

(3)

(b) Use the iterative formula

$$x_{n+1} = \sqrt[3]{\frac{4(3x_n^{\,2} + 2)}{x_n^{\,2} + 1}}$$

with $x_0 = 2$, to find x_1, x_2 and x_3 giving your answers to 3 decimal places.

(3)

The equation $f(x) = 0$ has a single root, α.

(c) By choosing a suitable interval, prove that $\alpha = 2.247$ to 3 decimal places.

(2)

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Question 1 continued

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Question 1 continued

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Question 1 continued

Q1

(Total 8 marks)

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2. The curve C has equation

$$y^3 + x^2y - 6x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence find the exact coordinates of the points on C for which $\frac{dy}{dx} = 0$

(6)

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Question 2 continued

(Total 11 marks)

Q2

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3. The number of bacteria in a liquid culture is modelled by the formula

$$N = 3500(1.035)^t, \quad t \geqslant 0$$

where N is the number of bacteria t hours after the start of a scientific study.

(a) State the number of bacteria at the start of the scientific study.

(1)

(b) Find the time taken from the start of the study for the number of bacteria to reach 10 000
Give your answer in hours and minutes, to the nearest minute.

(4)

(c) Use calculus to find the rate of increase in the number of bacteria when $t = 8$
Give your answer, in bacteria per hour, to the nearest whole number.

(3)

Leave blank

Question 3 continued

Q3

(Total 8 marks)

Leave blank

4. (a) Prove that

$$\frac{1-\cos 2x}{\sin 2x} \equiv \tan x, \qquad x \neq \frac{n\pi}{2}$$

(3)

(b) Hence solve, for $0 \leqslant \theta < 2\pi$,

$$3\sec^2\theta - 7 = \frac{1-\cos 2\theta}{\sin 2\theta}$$

Give your answers in radians to 3 decimal places, as appropriate.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

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Question 4 continued

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Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 9 marks)

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5. (i) Find

$$\int \left((3x + 5)^9 + e^{5x} \right) dx$$

(3)

(ii) Given that b is a constant greater than 2, and

$$\int_2^b \frac{x}{x^2 + 5} dx = \ln\left(\sqrt{6}\right)$$

use integration to find the value of b.

(5)

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Question 5 continued

Q5

(Total 8 marks)

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6.

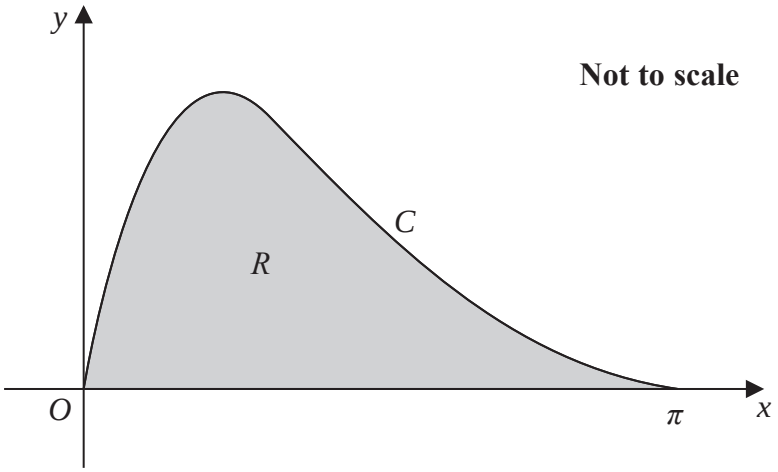

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 2e^{-x}\sqrt{\sin x}$, $0 \leqslant x \leqslant \pi$.
The finite region R, shown shaded in Figure 1, is bounded by the curve and the x-axis.

(a) Complete the table below with the value of y corresponding to $x = \frac{\pi}{2}$, giving your answer to 5 decimal places.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y	0	0.76679		0.15940	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of the region R. Give your answer to 4 decimal places.

(3)

(c) Given $y = 2e^{-x}\sqrt{\sin x}$, find $\frac{dy}{dx}$ for $0 < x < \pi$.

(3)

The curve C has a maximum turning point when $x = a$.

(d) Use your answer to part (c) to find the value of a, giving your answer to 3 decimal places.

(3)

Leave blank

Question 6 continued

Leave blank

Question 6 continued

Leave blank

Question 6 continued

(Total 10 marks)

Q6

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7. (a) Use the binomial series to expand

$$\frac{1}{(2-3x)^3} \qquad |x| < \frac{2}{3}$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

$$f(x) = \frac{4+kx}{(2-3x)^3} \qquad \text{where } k \text{ is a constant and } |x| < \frac{2}{3}$$

Given that the series expansion of $f(x)$, in ascending powers of x, is

$$\frac{1}{2} + Ax + \frac{81}{16}x^2 + \cdots$$

where A is a constant,

(b) find the value of k,

(2)

(c) find the value of A.

(2)

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Leave blank

Question 7 continued

Q7

(Total 9 marks)

Leave blank

8. Use partial fractions, and integration, to find the exact value of $\int_3^4 \frac{2x^2 - 3}{x(x-1)} \mathrm{d}x$

Write your answer in the form $a + \ln b$, where a is an integer and b is a rational constant.

(8)

Leave blank

Question 8 continued

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Question 8 continued

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Question 8 continued

Q8

(Total 8 marks)

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9.

$$f(x) = 2\ln(x) - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(5)

(b) Find the exact solutions of the equation $|f(x)| = 4$

(4)

$$g(x) = e^{x+5} - 2, \quad x \in \mathbb{R}$$

(c) Find $gf(x)$, giving your answer in its simplest form.

(3)

(d) Hence, or otherwise, state the range of gf.

(1)

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Question 9 continued

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Question 9 continued

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Question 9 continued

Q9

(Total 13 marks)

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10.

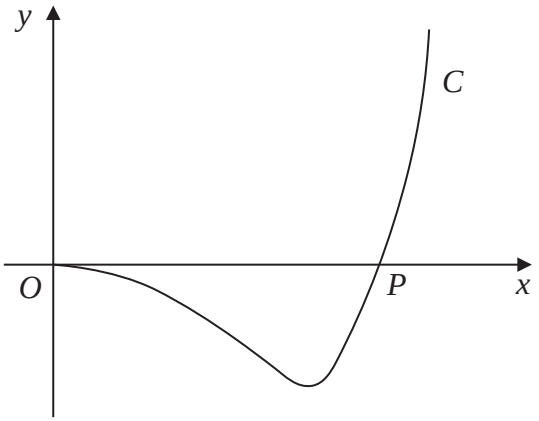

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = \frac{20t}{2t+1} \qquad y = t(t-4), \quad t > 0$$

The curve cuts the x-axis at the point P.

(a) Find the x coordinate of P.

(2)

(b) Show that $\frac{dy}{dx} = \frac{(t-A)(2t+1)^2}{B}$ where A and B are constants to be found.

(5)

(c) (i) Make t the subject of the formula

$$x = \frac{20t}{2t+1}$$

(ii) Hence find a cartesian equation of the curve C. Write your answer in the form

$$y = f(x), \qquad 0 < x < k$$

where $f(x)$ is a single fraction and k is a constant to be found.

(6)

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Question 10 continued

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Question 10 continued

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Question 10 continued

(Total 13 marks)

Q10

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11. (a) Given $0 \leqslant h < 25$, use the substitution $u = 5 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{5 - \sqrt{h}} = -10\ln\left(5 - \sqrt{h}\right) - 2\sqrt{h} + k$$

where k is a constant.

(6)

A team of scientists is studying a species of tree.

The rate of change in height of a tree of this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.2}\left(5 - \sqrt{h}\right)}{5}$$

where h is the height of the tree in metres and t is the time in years after the tree is planted.

One of these trees is 2 metres high when it is planted.

(b) Use integration to calculate the time it would take for this tree to reach a height of 15 metres, giving your answer to one decimal place.

(7)

(c) Hence calculate the rate of change in height of this tree when its height is 15 metres. Write your answer in centimetres per year to the nearest centimetre.

(1)

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Question 11 continued

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Question 11 continued

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Question 11 continued

Q11

(Total 14 marks)

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12. Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 8 \\ 4 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters.

The lines l_1 and l_2 intersect at the point A.

(a) Write down the coordinates of A.

(1)

Given that the acute angle between l_1 and l_2 is θ,

(b) show that $\sin\theta = k\sqrt{2}$, where k is a rational number to be found.

(5)

The point B lies on l_1 where $\lambda = 4$

The point C lies on l_2 such that $AC = 2AB$.

(c) Find the exact area of triangle ABC.

(3)

(d) Find the coordinates of the two possible positions of C.

(5)

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Question 12 continued

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Question 12 continued

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Question 12 continued

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Question 12 continued

Q12

(Total 14 marks)

TOTAL FOR PAPER: 125 MARKS

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